Soluciones Ejercicios de Integrales
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Nota. A todos las primitivas hay que sumarle las constante de integración k.

Pagina 194.

1. $\int 1 = x$
2. $\int 2x = x^2$
3. $\int x = \frac{x^2}{2}$
4. $\int 3x^2 = x^3$
5. $\int x^2 = \frac{x^3}{3}$
6. $\int x^5 = \frac{1}{6}x^6 = \frac{1}{6}x^6$
7. $\int (k+1)\, x^k = x^{k+1}$
8. $\int x^k = \frac{x^{k+1}}{k+1} + c$
9. $\int \sqrt[3]{3x^2} = \sqrt[3]{3} \int x^{\frac{2}{3}} = \sqrt[3]{3}\frac{3}{5}x^{\frac{5}{3}}$
10. $\int \sqrt{x^3} = \frac{2}{5}x\sqrt{(x^3)}$
11. $\int (3x^3 - 7x - 2) = \frac{3}{4}x^4 - \frac{7}{2}x^2 - 2x$
12. $\int \frac{1}{x} = \ln x$
13. $\int \frac{1}{x-5} = \ln (x-5)$
14. $\int \frac{5}{x} = 5 \ln x$
15. $\int \frac{3}{x+4} = 3 \ln (x+4)$
16. $\int \cos x = \sin x$
17. $\int \cos 2x = \frac{1}{2} \sin 2x$
18. $\int 5 \cos (3x+1) = \frac{5}{3} \sin (3x+1)$
19. $\int \sin x = -\cos x$
20. $\int \sin x . \cos x = \frac{1}{2} \sin^2 x$
21. $\int \frac{1}{1+x^2} = \arctan x$
22. $\int \cos x^2 . 2x = \sin x^2$
23. $\int \cos (x^2+1)\, .2x = sen\, (x^2+1)$
24. $\int \sqrt{x-5} = \frac{2}{3}\left(\sqrt{(x-5)}\right)^3$
25. $\int \sqrt{x^2+2}.2x = \frac{2}{3}\left(\sqrt{(x^2+2)}\right)^3$
26. $\int \frac{3x^2-2x}{x^3-x^2+3} = \ln (x^3 - x^2 + 3)$
27. $\int e^{x-5} = e^{x-5}$

Pagina 197

1. $\int (x^2+3x) = \frac{1}{3}x^3 + \frac{3}{2}x^2$
2. $\int (6x^2+3x+3) = 2x^3 + \frac{3}{2}x^2 + 3x$
3. $\int (x^3 - 4x + 2) = \frac{1}{4}x^4 - 2x^2 + 2x$
4. $\int (x^5 + 4x) = \frac{1}{6}x^6 + 2x^2$
5. $\int (x-1)^3 = \frac{1}{4}(x-1)^4$
6. $\int (4x^3 - 7x^2 + 5x + 1) = x^4 - \frac{7}{3}x^3 + \frac{5}{2}x^2 + x$
7. $\int x^{\frac{2}{5}} = \frac{5}{7}\left(\sqrt[5]{x}\right)^7$
8. $\int \sqrt[3]{x} = \frac{3}{4}\left(\sqrt[3]{x}\right)^4$
9. $\int x^{\frac{-6}{7}} = 7\sqrt[7]{x}$
10. $\int \frac{1}{\sqrt[7]{x^5}} = \frac{7}{2}\frac{x}{\sqrt[7]{(x^5)}}$
11. $\int \frac{2}{x} = 2 \ln x$
12. $\int \sqrt{3x} = \frac{2}{3}\left(\sqrt{x}\right)^3\sqrt{3}$
13. $\int (x^2+4x)(x^2-1) = \frac{1}{5}x^5 + x^4 - \frac{1}{3}x^3 - 2x^2$
14. $\int (x^5 - 3x^4 + 5x^2 - 1) = \frac{1}{6}x^6 - \frac{3}{5}x^5 + \frac{5}{3}x^3 - x$
15. $\int (x^2 - 3x) = \frac{1}{3}x^3 - \frac{3}{2}x^2$
16. $\int (4x^2 - 5x + 7) = \frac{4}{3}x^3 - \frac{5}{2}x^2 + 7x$
17. $\int (x - \sin x) = \frac{1}{2}x^2 + \cos x$
18. $\int (x + \cos x) = \frac{1}{2}x^2 + \sin x$
19. $\int (x - 2\tan x) = \frac{1}{2}x^2 + 2\ln(\cos x)$
20. $\int (\sin x + e^x) = -\cos x + e^x$
21. $\int (\cos x - 5e^x) = \sin x - 5e^x$
22. $\int (e^x - e^{-x}) = e^x + e^{-x}$
23. $\int (e^x + 3e^{-x}) = e^x - 3e^{-x}$
24. $\int \frac{3}{x-5} = 3 \ln (x-5)$
25. $\int \frac{1}{2x+7} = \frac{1}{2} \ln (2x+7)$
26. $\int \frac{3}{1+x^2} = 3 \arctan x$
27. $\int \frac{x^2-5x+4}{x+1} = \frac{1}{2}x^2 - 6x + 10 \ln (x+1)$

Pagina 201 ejercicio 1,

a) $\int \frac{2x-5}{x^2-5x+6}dx = \ln(x^2-5x+6)$
b) $\int \sin x \cos^4 x \, dx = -\frac{1}{5}\cos^5 x$
c) $\int \sin\left(\frac{x}{2}\right)dx = -2\cos\frac{1}{2}x$
d) $\int \frac{-3x^3}{1+x^4}dx = -\frac{3}{4}\ln(1+x^4)$
e) $\int \frac{x^2}{1+x^3}dx = \frac{1}{3}\ln(1+x^3)$
f) $\int \frac{dx}{(x+2)^3} = -\frac{1}{2(x+2)^2}$
g) $\int \tan x dx = -\ln(\cos x)$
h) $\int e^{7x}dx = \frac{1}{7}e^{7x}$
i) $\int e^{-2x+3}dx = -\frac{1}{2}e^{-2x+3}$
j) $\int xe^{x^2}dx = \frac{1}{2}e^{x^2}$
k) $\int x^2e^{x^3}dx = \frac{1}{3}e^{x^3}$
l) $\int \cos 3x dx = \frac{1}{3}\sin 3x$
m) $\int x\sin(x^2+4)\,dx = -\frac{1}{2}\cos(x^2+4)$
n) $\int \frac{2x}{1+x^4}dx = \arctan x^2$

Pagina 203 Ejercicio 1

a) $\int \left(3\sqrt[3]{x} + \frac{\sqrt{x}}{x^3}\right)dx = \frac{9}{4}\left(\sqrt[3]{x}\right)^4 - \frac{2}{3\left(\sqrt{x}\right)^3}$
b) $\int \frac{x^3-7x^2+4}{x}dx = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 4\ln x$
c) $\int (x^2-1)^2\,dx = \frac{1}{5}x^5 - \frac{2}{3}x^3 + x$
d) $\int \left(\frac{1}{x^2} + \frac{1}{x^2+1}\right)dx = -\frac{1}{x} + \arctan x$
e) $\int \left(\sqrt{x} + \frac{1}{x}\right)^2 dx = \frac{1}{2}x^2 + 4\sqrt{x} - \frac{1}{x}$
f) $\int \left(\frac{2}{x+4} + \frac{2}{x-4}\right)dx = 2\ln(x+4) + 2\ln(x-4)$
g) $\int \frac{e^x}{e^x+4}dx = \ln(e^x+4)$
h) $\int \frac{x^3-5x+3}{x-2}dx = \frac{1}{3}x^3 + x^2 - x + \ln(x-2)$
i) $\int \frac{3x+2}{\sqrt{x}}dx = 2\left(\sqrt{x}\right)^3 + 4\sqrt{x}$
j) $\int \frac{\pi}{2}\sin \pi x \, dx = -\frac{1}{2}\cos \pi x$
k) $\int \frac{1-\cos x}{2}dx = \frac{1}{2}x - \frac{1}{2}\sin x$
l) $\int \frac{2}{\sin^2 x}dx = -\frac{2}{\sin x}\cos x$
m) $\int \frac{1+2x}{1+x^2}dx = \ln(1+x^2) + \arctan x$
n) $\int \frac{1-\sin^2 x}{\cos x}dx = \sin x$

Ejercicio 2

a) $\int \frac{3}{(2x-5)^4}dx = -\frac{1}{2(2x-5)^3}$
b) $\int (2x-7)^{10}\,dx = \frac{1}{22}(2x-7)^{11}$
c) $\int x\sqrt{2x^2+17}dx = \frac{1}{6}\left(\sqrt{(2x^2+17)}\right)^3$
d) $\int \frac{4x}{\sqrt[3]{x^2-8}}dx = 3\left(\sqrt[3]{(x^2-8)}\right)^2$
e) $\int \sin^3 x \cos x dx = \frac{1}{4}\sin^4 x$
f) $\int \frac{1}{\sqrt{3x+2}}dx = \frac{2}{3}\sqrt{(3x+2)}$
g) $\int e^{\sin x}\cos x dx = e^{\sin x}$
h) $\int \frac{\ln x}{x}dx = \frac{1}{2}\ln^2 x$
i) $\int \sin 2x \cos 2x dx = -\frac{1}{4}\cos^2 2x$
j) $\int \frac{x+1}{\sqrt[3]{x^2+2x+12}}dx = \frac{3}{4}\left(\sqrt[3]{(x^2+2x+12)}\right)^2$
k) $\int \frac{\tan x}{\cos^2 x}dx = \frac{1}{2\cos^2 x}$
l) $\int \frac{\arctan x}{1+x^2}dx = \frac{1}{2}\arctan^2 x$

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Ejercicio 4

a) $\int \frac{e^x}{e^x-1}dx = \ln(e^x-1)$
b) $\int x^2 \sin x^3 dx = -\frac{1}{3}\cos x^3$
c) $\int \frac{x}{\sqrt{x^2+1}}dx = \sqrt{(1+x^2)}$
d) $\int \frac{x^2-4}{x-3}dx = \frac{1}{2}x^2 + 3x + 5\ln(x-3)$
e) $\int \frac{2+x}{x^2+1}dx = \frac{1}{2}\ln(1+x^2) + 2\arctan x$
f) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = -2\cos\sqrt{x}$
g) $\int \frac{x^2-x+7}{x+2}dx = \frac{1}{2}x^2 - 3x + 13\ln(2+x)$
h) $\int \frac{1}{4+x^2}dx = \frac{1}{2}\arctan\frac{1}{2}x$
i) $\int \frac{x}{\sqrt{1-x^2}}dx = -\sqrt{(1-x^2)}$
j) $\int \frac{1}{\sqrt{4-x^2}}dx = \arcsin\frac{1}{2}x$
k) $\int \frac{x}{1+x^4}dx = \frac{1}{2}\arctan x^2$
l) $\int \frac{3x+1}{x^2+9}dx = \frac{3}{2}\ln(x^2+9) + \frac{1}{3}\arctan\frac{1}{3}x$

Ejercicio 5

a) por partes
b) $\int xe^x dx = xe^x - e^x$
c) $\int (e^{2x} + e^{-2x})\,dx = \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x}$
d) por partes
e) $\int \frac{2}{3+x^2}dx = \frac{2}{3}\sqrt{3}\arctan\frac{1}{3}x\sqrt{3}$
f) $\int \frac{2x}{4+x^4}dx = \frac{1}{2}\arctan\frac{1}{2}x^2$

Integrales definidas y área

Pagina 217

1) $\int_1^3 (x^2+1)\,dx = \frac{32}{3}$

2) $\int_0^{\frac{\pi}{2}} \cos x dx = 1$

3) $\int_0^1 e^x dx = e-1$

4)

a) $\int \frac{3x^4+x+4}{\sqrt{x}} dx = \frac{2}{3}\left(\sqrt{x}\right)^9 + \frac{2}{3}\left(\sqrt{x}\right)^3 + 8\sqrt{x} + c,$ $c = 350,6$

b) $\int \frac{3x^4+7x^3-3}{x+2} dx = \frac{3}{4}x^4 + \frac{1}{3}x^3 - x^2 + 4x - 11\ln(x+2) + c,$ $c = -\frac{724}{3} + 11\ln 6$

c) $\int \frac{\sqrt{3x}\sqrt[3]{x^2}+\sqrt{3}}{\sqrt{3x^2}} dx = \frac{1}{7}\left(6\left(\sqrt[6]{x}\right)^7 + 7\ln x\right)\frac{x}{\sqrt{(x^2)}} + c$ $c = -\frac{384}{7}\sqrt[6]{64} - \ln 64$

d) $\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{(1-x^2)} + c$ $c = 0.8$

e) $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$ $c = -\frac{\pi}{6}$

5)

a) $\int \frac{x}{\sqrt{1-x^2}} dx = -\sqrt{(1-x^2)}$ $c = 0.4$

b) $\int \frac{3x^4-2x^3+5}{x-2} dx = \frac{3}{4}x^4 + \frac{4}{3}x^3 + 4x^2 + 16x + 37\ln|(x-2)|$ $F(0) = .37\ln|(0-2)| = 25,646$

6) 12 unidades de área.

7)

a) $\int_0^3 (x^3-7x)\,dx = -\frac{45}{4}$

b) $\int_2^6 \frac{1}{x} dx = \ln 3$

c) $\int_0^{2\pi} \sin x dx = 0$

d= $\int_2^6 \sqrt{x-2} dx = \frac{16}{3}$